Executive Summary

This review examines the health of research in the mathematical sciences in Australia. The review also investigates the provision of high level mathematical services, and demonstrates how the nation gains benefit from its investment in this discipline.

Mathematics is the study of measurement, forms, patterns, variability and change. It evolved from our efforts to understand the natural world. Its roots go too far back in prehistory to trace, but an unbroken chain of development has continued for more than two thousand years since Greek civilisation at the time of Euclid. The end of this chain, modern mathematical science, is in its own right a supreme creation of the human intellect; it is also critical for economic competitiveness, and a basis for investigations in many fields.

Over the course of time, the mathematical sciences have developed a rich and intrinsic culture that feeds back into the natural sciences and technology, often in unexpected ways. The mathematical sciences now reach far beyond the physical sciences and engineering; they reach into medicine, commerce, industry, the life sciences, the social sciences, and to every other application that needs quantitative analysis. Their influence has been vastly enlarged by the advent of modern computers. Computer use in problem solving, simulation and decision making relies on powerful computational algorithms derived from new mathematical developments.

At the launch of a major international meeting a few years ago, the then President of France spoke in the context of mathematics of "la necessité d'une politique de la science soucieuse du long terme, attentive à l'équilibre entre recherche, enseignement, économie"[1]. In our words, he is asserting that there is interdependence between research within the mathematical sciences, the application of these sciences in other disciplines, and their teaching. Each of these nourishes the others with ideas, methods and inspiration.

Research in the mathematical sciences used to be primarily undertaken on an individual basis. Collaborative research has now become more widespread, perhaps to the extent that it is more common than individual research. Taken together, individuals and groups pursuing research in the mathematical sciences constitute a human and intellectual resource of utmost national significance.

[1] F. Mitterand, *message for the colloquium "Future Mathematics"* (December, 1987) pointed to "the need for a politics of science aware of the long term, and conscious of a correct balance of research, teaching and economic factors".

This review is also concerned with Australia's performance in the delivery of high level mathematical services. The review finds unequivocal evidence that, as an economic and social instrument, advanced mathematical services relying on the mathematical sciences are critically important to Australia.

The mathematical sciences are universal. Some of the words above are taken from the National Policy Statement of the American Mathematical Society, yet their relevance to the Australian context is manifest. Elsewhere, we make extensive use of documents developed by the Society for Industrial and Applied Mathematics in the USA.

The present Strategic Review of Mathematical Sciences Research and Advanced Mathematical Services in Australia has four principal findings and 20 recommendations.

Principal Findings

1. It is essential for Australia to have a sound research base in the mathematical sciences for the following reasons:

- to be able to respond to new research ideas and opportunities
- to capture benefit through collaborative research and downstream technology transfer
- to educate future mathematical sciences graduates
- to contribute to the economic and cultural strength of the nation
- to benefit from international developments

In general, Australia possesses a sound research base, although certain sub-disciplines, among them operations research and financial mathematics, need to be strengthened.

2. The mathematical sciences are critical to Australia's economic competitiveness and quality of life, and will become more so. The mathematical sciences are generic and enabling technologies. They are essential to the prosperity of many value-adding industries in Australia.[2]

[2]These remarks endorse and confirm the relevance to the Australian context of the findings of a major US-sourced report on the importance of the mathematical sciences to modern economies: *Mathematical Sciences, Technology and Economic Competitiveness*, James G Glimm (ed.), Board on Mathematical Sciences, National Research Council, National Academy Press, Washington, DC, 1991.

3. The mathematical sciences make a vital contribution to many fields of research and endeavour. The importance of this contribution needs further emphasis because

- much work in the mathematical sciences is multi-disciplinary in nature
- there is a spillover of concepts and techniques from the mathematical sciences into other disciplines, particularly through methods and software widely used in those disciplines
- researchers in many other disciplines (including the social sciences) who would not describe themselves as mathematical scientists nonetheless make extensive use of mathematical and statistical concepts

4. The mathematical sciences profession in Australia faces a number of major challenges:

- improving the image of the profession to match its importance and effectiveness
- balancing an age distribution which is currently skewed by the growth in the profession in the late 1960s and 1970s
- redressing the gender imbalance at senior levels
- attracting good undergraduate students into mathematical sciences courses
- increasing opportunities for postdoctoral level researchers
- broadening the funding base for research
- educating potential users to the value of the mathematical sciences
- improving technology transfer programs and associated educational programs, particularly for SMEs (small to medium enterprises)

If these challenges are not addressed successfully, there will be significant diminution in Australia's capabilities in the mathematical sciences, to the detriment of the nation.

Recommendations

A brief justification or background statement precedes each recommendation. Further details for each recommendation are given in the bulk of the report.

The nature of mathematical sciences research is continually changing. One notable feature is that sub-disciplines of mathematics that previously were investigated without applications in mind now have important applications. An obvious example is the use of number theory and group theory in cryptography. The old usage of the terms 'pure' and 'applied' mathematics is inappropriate and a source of confusion.

Recommendation 1

- ☐ In addition to the Field of Research classification, research activities in the mathematical sciences should be described by the Australian Standard Research (Type of Activity) Classification, namely basic research (pure or strategic), applied research and experimental development.

[**Mathematical sciences researchers, Heads of Department, Deans**]

In submissions to the Review and through its site visits, the Working Party formed the view that there was a significant weakness in the research framework underpinning Operations Research (see footnote 2 in Chapter 1 for a definition). The Australian Research Council has a variety of mechanisms which can address this situation.

Recommendation 2

- ☐ The Australian Research Council is encouraged to designate the field of Operations Research as a priority area for ARC grants, particularly as a Key Centre of Teaching and Research.

[**ARC**]

The Working Party believes that the nation's mathematical scientists have much to gain by engaging more fully with SE Asia and the Pacific Rim. Benefits are expected for both basic and applied research.

Recommendation 3

- ☐ To promote Australia's capabilities, mathematical sciences researchers are encouraged to give higher priority than is now customary to participation in scientific conferences in SE Asia and the Pacific Rim.

[**Mathematical sciences researchers, Professional Societies**]

It is essential that some consulting work be undertaken by some mathematical sciences departments. This consulting work should be viewed in a similar light to service teaching, namely that it is an important part of the profession. Consulting encourages the transfer of mathematical sciences technology to users outside the universities and thereby helps to develop a culture of innovation in Australia. It also offers a valuable opportunity to diversify funding sources. In view of these benefits, consulting should be appropriately managed and rewarded.

Recommendation 4

- ☐ Universities should enhance their mechanisms for recruiting and rewarding academic staff who, through consulting and similar activities, provide advanced mathematical services to external customers. Consulting should be fully costed, and it should be managed through departments and not on an individual basis.

[**AVCC, Deans**]

For their continued well-being, the mathematical sciences are dependent on inter-disciplinary collaboration and technology transfer to users. Mathematical sciences departments should establish mechanisms to receive external advice about their courses and prospective activities. We therefore recommend that

Recommendation 5

- ☐ All mathematical sciences departments should have external advisory mechanisms to assist in the development of strategic objectives, to inform the department about research opportunities, and to get external feedback on the suitability of existing and proposed courses.

[**Heads of Department, Deans**]

The mechanisms by which mathematical sciences technology is transferred to users are weak. This adversely affects the culture of innovation in Australia. Thus it is important that existing mechanisms be exploited, existing highlights acknowledged and new mechanisms entertained.

Recommendation 6a

- ☐ Academic mathematical scientists are encouraged to bid more vigorously for ARC Collaborative Research Awards and Australian Postgraduate Awards (Industry).

[**Mathematical scientists**]

Recommendation 6b

- ☐ The Mathematics-in-Industry Study Group should be continued, preferably as part of the activities of a CRC for Industrial Applications of the Mathematical Sciences. Should such a CRC not be established, then the Australian Mathematical Society (through ANZIAM) is encouraged to co-ordinate ongoing arrangements for the Study Group.

[**Chief Scientist, ANZIAM**]

Recommendation 6c

- ☐ Mathematical scientists are encouraged to communicate more effectively with the media and general public.

[**Mathematical scientists**]

Recommendation 6d

- ☐ CSIRO is encouraged to continue its funding support for the activities of CSIRO Division of Mathematics and Statistics. This Division should continue to operate on a disciplinary basis.

[**CSIRO**]

Recommendation 6e

- ☐ DIST is encouraged to develop a specialist program so that small to medium enterprises (SMEs) have access to advanced mathematical services. The benefits of this program should be communicated. SMEs need to be assured of access to the 150% taxation benefit for advanced mathematical services provided to them.

[**DIST**]

Recommendation 6f

- ☐ Academics are encouraged to seek secondments to industry during periods of study leave. Where payments are made by industry for these secondments, taxation relief should be available to the employers.

[Mathematical sciences researchers, ATO]

Australian mathematics is greying. The age distribution of staff in mathematical sciences departments is heavily skewed towards the 45 and upwards group. If retirement continues to take place by age 65 as at present, there will be a large number of retirements over the 10–15 year scope of this review. Alternatively, if non-compulsory retirement becomes the norm across Australia, then the number of retirements is likely to be less, but replenishment of the profession by young researchers will be diminished. In either case, the profession faces a major challenge because of the present age structure of departments.

Recommendation 7

- ☐ Universities must develop plans to address difficulties caused by the present age structure in mathematical sciences departments. Consideration should be given to attractive early retirement plans and mechanisms for retaining promising postdoctoral researchers and grooming future leaders in the profession.

[AVCC, Deans, Heads of Department]

The mathematical sciences in Australia, as in most parts of the world, suffer from a chronic gender imbalance at senior levels. It is important to encourage talented female students to continue in the profession. The review found that talented female students need different sorts of support at different stages of their career: on entering university, after an undergraduate degree, and then as a postdoctoral researcher or junior academic.

Recommendation 8a

- ☐ Professional societies, academic departments and employers of mathematical sciences graduates are encouraged to promote activities, aimed at senior high school students and senior undergraduates, which demonstrate career opportunities for talented female mathematical sciences students and which encourage them to continue to further studies.

[Heads of Department, Professional Societies]

Recommendation 8b

- ☐ The ARC is encouraged to award a special two year postdoctoral award to provide a role model for female mathematical scientists. The award, which would need to be appropriately publicised, might be called the Hanna Neumann Postdoctoral Fellowship.

[**ARC**]

Recommendation 8c

- ☐ Employers are encouraged to provide flexible arrangements so that female researchers at postdoctoral or junior academic level can continue their careers after breaks for child-rearing.

[**ARC, AVCC**]

The mobility of mathematical sciences researchers between institutions is low by international standards, even allowing for the fact that Australia is a large, sparsely populated country. Greater mobility of researchers between institutions will improve the skills base and knowledge in the profession. We urge the introduction of mechanisms to improve the mobility of junior researchers.

Recommendation 9a

- ☐ Financial disincentives to mobility between institutions of postgraduates and postdoctoral fellows should be removed by provision of increased removal and travel allowances, and in other ways to be identified. Mathematical sciences departments should actively recruit postgraduates and postdoctoral fellows from other institutions.

[**ARC, Heads of Department**]

Recommendation 9b

- ☐ The professional societies are encouraged to facilitate access to information from university departments concerning possible PhD topics and supervision arrangements. Departments are encouraged to maintain such information on their World Wide Web pages, and the societies through their own WWW server should provide easy links to those pages and should publicise that fact.

[**Professional Societies, Heads of Department**]

There is a low awareness in industry of the contribution of the mathematical sciences to the economic competitiveness of the nation. In part, this results from the fact that mathematical sciences students do not have sufficient education and training in important skills such as

communication, project work on industrial case studies, and collaboration in teams. We consider that a postgraduate diploma or master's level course is required to acquire these skills. Moreover, the act of talking to prospective employers and students will generate increased awareness.

Recommendation 10

- ☐ Departments are encouraged to carry out market research aimed at establishing master's level courses which will meet the needs of Australian industry. These courses should embody necessary mathematical, statistical and computational knowledge, communication skills, management methods, industrial placements and project work. As appropriate, these courses should be established.

[AVCC, Deans, Heads of Department]

The mathematical sciences now depend critically on computers for research and communication with colleagues. Moreover the mathematical sciences have become laboratory based; as such, funding for mathematical sciences departments should be on a comparable basis to computer science departments.

Recommendation 11

- ☐ Departments and universities should ensure that staff and students have access to appropriate computers, software, support staff and network connections. To support such infrastructure for advanced teaching, research and communications, mathematical sciences departments should be funded on a comparable basis to computer science departments.

[DEET, AVCC, Deans]

Developments in information technology will have a profound effect on the dissemination of knowledge in all disciplines, including the mathematical sciences. Within the time horizon of this review, journal publishing is almost certain to become largely electronic in nature. In addition, electronic developments like the World Wide Web will enable departments to promote their activities, increase the effectiveness of their teaching, and disseminate knowledge created by staff. It is impossible to be precise about details of the changes to be created by information technology developments, but it is possible to develop a mindset that will enable the changes to be relatively smooth and productive.

Recommendation 12a

- ☐ It is of utmost importance that networks be capable of handling the future challenges of the information age.

[**AVCC**]

Recommendation 12b

- ☐ Professional societies in the mathematical sciences are encouraged to continue their development of electronic operations. Mathematical sciences departments are encouraged to use the World Wide Web and its successors to promote their activities, to disseminate knowledge, and to increase the effectiveness of their teaching and research. Funding bodies such as the ARC are encouraged to provide funding to build up appropriate information technology infrastructure.

[**ARC, Professional Societies, Heads of Department, Deans**]

Hardware and software developments mean that the mathematical sciences have become laboratory based, and that computational science has become a third strand of scientific endeavour along with theory and experiment. Computer developments will have major ramifications on the way that the mathematical sciences are taught: for example, symbolic manipulation packages reduce the need for human involvement in algebraic manipulations, and packages enable routine solution of sets of ordinary differential equations. It is highly likely that further major changes to courses will take place over the 10–15 year horizon of this review.

Recommendation 13

- ☐ Mathematical sciences departments are urged to survey their courses and re-design them to make best use of the growing power of computers.

[**Heads of Department, Deans**]

As noted above, we see that computer developments will have a profound effect on the way that university mathematics is taught in the future. A 'universal acquisition' policy will help departments cater for future needs of students in an efficient way.

Recommendation 14

- ☐ The government is encouraged to introduce a funding scheme (*e.g.* of HECS type or by way of low cost loans, and/or by sales tax exemption) by which all mathematical sciences students can readily acquire a suitable computer, software and modem connections. Universities and departments should support this recommendation by providing appropriate software and network connections, and by using bulk-buying power to obtain low prices for hardware and software.

[**DEET, AVCC, Deans, Heads of Department**]

The mathematical sciences have a major effect on multi-disciplinary research and on the economic competitiveness of Australia. Nevertheless it is sometimes difficult to ascertain just how far the mathematical sciences have penetrated into industry and other fields. In this context, we make the following two recommendations which will assist policy formulation.

Recommendation 15

- ☐ The Department of Industry, Science and Technology should introduce Field of Research classifications into applications for the 150% tax concession for industrial Research and Development.

[**DIST**]

Recommendation 16

- ☐ The Australian Mathematical Society is encouraged to collaborate with the other professional societies to maintain
  - a register of ARC Large Grants awarded for mathematical sciences research
  - a register of PhD students in the mathematical sciences with information including topic, supervisor, funding support, gender

[**Australian Mathematical Society**]

The funding base for research in the mathematical sciences is relatively narrow. It consists primarily of ARC grants and funds generated by service teaching. Although the mathematical sciences community has been successful in winning an appropriate share of ARC funds, the profession would benefit by broadening the funding base for research.

Recommendation 17

- ☐ To broaden the funding base for the profession, mathematical sciences researchers are encouraged to apply for funds from a wide range of sources including
  - ARC collaborative grants scheme
  - APA (Industry) collaborative grants scheme
  - the ARC Key Centres program
  - competitive research grants from the Industry R&D Board of DIST
  - the DIST Bilateral Science and Technology Program (for international collaboration)
  - industry association funds
  - other government programs including NHMRC funds

[**Academic researchers, Heads of Department, Deans**]

Service teaching is a very important activity for mathematical sciences departments. It contributes to the development of inter-disciplinary collaboration and provides funds to maintain the department and its research programs.

Recommendation 18

- ☐ Departments must recognise the role that service teaching plays in maintaining the mathematical level of other disciplines and fostering links with those disciplines. Departments should be attentive to nurturing service teaching arrangements and meeting the needs of client disciplines. Universities should beware of fragmenting the mathematical sciences through devolution of service teaching.

[**Heads of Department, Deans, AVCC**]

Australia has a need for a special research centre to enrich basic research in the mathematical sciences and thereby contribute to the framework on which applied research can be built. To provide the flexibility to address changing needs, particularly rapidly developing opportunities, this Centre should have no permanent scientific staff, and should rely on visiting scholars to undertake research programs.

Recommendation 19a

- ☐ The Australian Research Council should facilitate application under the SRC program by the mathematical sciences disciplines for a National Research Centre in the mathematical sciences.

Recommendation 19b

- ☐ The National Committee for Mathematics should conduct a competitive tender amongst universities prepared to offer funds to be the site of a proposed National Research Centre in the mathematical sciences in similar style to MSRI, IMA, the Fields Institute or the Newton Institute.

[**ARC, National Committee for Mathematics**]

In contrast with many disciplines, the mathematical sciences do not possess a specific industry sector which can provide funding and industrial collaboration. Consequently, there are structural difficulties in establishing high level collaborative activities and in winning major grants such as those for Collaborative Research Centres. Nevertheless, the evidence is compelling that the mathematical sciences are pervasive throughout industry and essential to the economic competitiveness of the nation. It is important to strengthen the mechanisms by which mathematical sciences technology is developed and transferred to users. We believe this will help to develop a culture of innovation in Australia. The CRC program offers a way to achieve these goals.

Recommendation 20

- ☐ The government should identify the mathematical sciences as an under-represented discipline in the CRC program and should therefore invite proposals to establish a CRC for Industrial Applications of the Mathematical Sciences.

[**DPMC, Chief Scientist**]